Kinetic modelling of runaway electrons

Tünde Fülöp

Theory and Simulation of Disruptions Workshop

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Adam Stahl
PhD student

Ola Embréus
PhD student

Eero Hirvijoki
Postdoc

Sarah Newton
Visiting researcher

István Pusztai
Assistant professor

Collaborators:

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Matt Landreman (Univ Maryland)
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Tools available for runaway studies at Chalmers

- Kinetics

  CODE – runaway electrons

  CODION – runaway ions

- Disruption modelling

  GO – 1D fluid code, consistent current and electric field evolution, atomic physics

  GO+CODE – with G Papp (IPP Garching)

- Radiation

  SYRUP – synchrotron spectra

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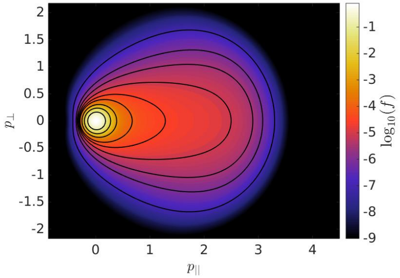
CODE (COllisional Distribution of Electrons)

Solves the kinetic equation for the electron distribution function

- 2D in momentum space, no spatial dependency
- Fully relativistic
- Runaway generation
  - Primary
  - Secondary
- Lightweight, continuum
- Very efficient steady-state solution



[Landreman, Stahl and Fülöp, CPC **185**, 847 (2014)]

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Recent improvements to CODE

- Synchrotron radiation reaction
- Bremsstrahlung radiation reaction
- Improved avalanche operators
- GO+CODE-related
  - Time-dependent plasma parameters
  - Momentum conserving collision operator
  - More flexible input-handling
  - Automatic grid extensions
- Full rewrite under way

Rosenbluth-Putvinski operator

- Knock-on collision = large-angle collision
- A runaway can transfer a large amount of momentum to another particle in one collision – can lead to avalanche
- If we let $p \to \infty$ for incoming particle, the source is

$$S_{\mathrm{RP}}(p,\xi) = \frac{n_r v_{\mathrm{rel}}}{4\pi \ln\Lambda} \delta(\xi - \xi_2) \frac{1}{p^2} \frac{\partial}{\partial p} \left(\frac{1}{1 - \sqrt{1+p^2}} \right)$$

[Rosenbluth and Putvinski, Nucl. Fusion **37**, 1355 (1997)]

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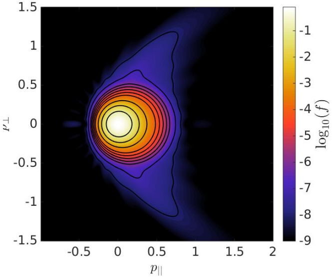
Problems:

- $\propto n_r$ – all runaways considered to have infinite momentum
- Secondary runaways can be generated with higher energy than any of the existing runaways!
- No change to incoming particle in collision – does not conserve particle number, energy or momentum



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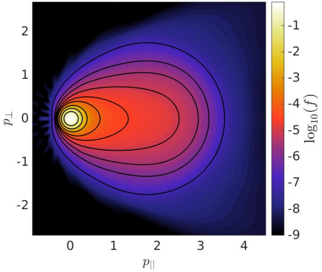

Chiu-Harvey operator

An improved operator is available!

$$S_{\mathrm{CH}}(p,\xi) \propto \frac{p_{\mathrm{in}}^4 f_{\xi=1}(p_{\mathrm{in}}) \sum(\gamma,\gamma_{\mathrm{in}})}{\gamma p \xi},$$

Σ is the Møller scattering cross-section
$f_{\xi=1}$ is pitch-angle averaged distribution

[S.C. Chiu, et al., Nucl. Fusion **38**, 1711 (1998),
R.W. Harvey et al., Phys. Plasmas. **7**, 4590 (2000)]

Improvements:

- Finite p_{in}
- Secondary particle momenta restricted by kinematics
- No δ-function in ξ

Unresolved:

- All incoming runaways have $\xi = 1$ $(\theta = 0)$
- No change to incoming particle after collision – not conservative

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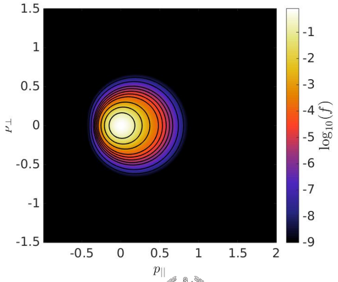

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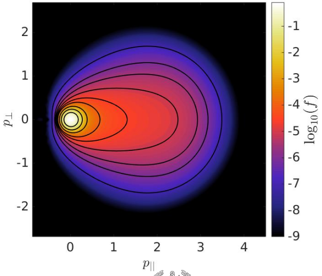

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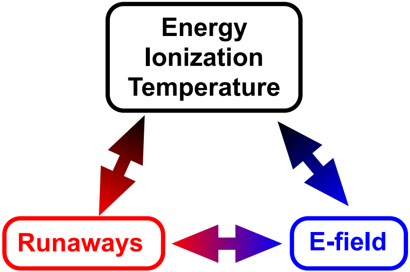
Impurity injection: GO

- 1D model for plasma cooling, runaway current and electric field evolution during impurity injection

- Energy balance equations for all species, including
  - Ohmic heating
  - Line radiation and Bremsstrahlung
  - Rate equations for ionization & recombination
  - Collisional energy exchange



[H Smith et al, PP **13** 102502 (2006);
K Gál et al, PPCF **50** 055006 (2008);
H Smith et al, PPCF **51** 124008 (2009);
T Fehér et al, PPCF **53** 035014, (2011);
G Papp et al, NF **53** 123017 (2013)]

Outline

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Critical field in E/E_c ramp-up

- Experiments show $E/E_c > 3-5$ needed for RE generation when ramping up E/E_c
  [Granetz, et al., Phys. Plasmas **21**, 072506 (2014),
  Paz-Soldan et al., Phys. Plasmas **21**, 022514 (2014)]
- We study the RE dynamics using CODE
- Two effects contribute to explain the observation

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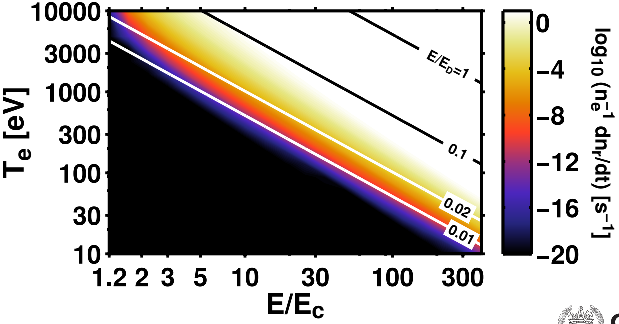
  - **Dreicer growth rate strongly T_e dependent** at fixed E/E_c
    - $E/E_D >$ 1%–2% is required for substantial growth



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  - **Dreicer growth rate strongly T_e dependent** at fixed E/E_c
    - $E/E_D > 1\%–2\%$ is required for substantial growth
  - **Synchrotron radiation reaction** leads to reduction in growth rate for small E/E_c
    - Synchrotron effects important for high T_e and low n_e
    - Runaway dynamics qualitatively different in disruption and flat-top scenarios

What about E/E_c ramp-down?

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Runaway growth-to-decay transition

- Build up RE tail, then ramp down E/E_c
- In experiments, visual synchrotron and HXR signals transitions from growth to decay at $E/E_c = 3$–5
  [Paz-Soldan et al., Phys. Plasmas **21**, 022514 (2014)]
- Simulations (including avalanche generation) show transition in RE growth at only slightly above E_c (~ 1.1)

BUT

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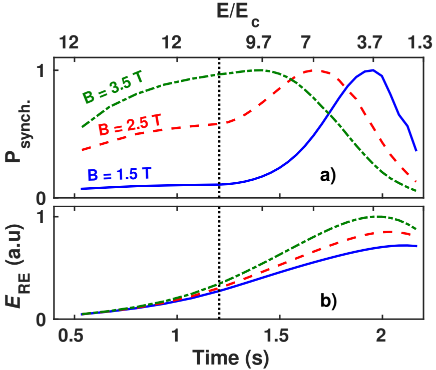
Runaway growth-to-decay transition

Synchrotron emission agrees with experiments!



[Stahl, Hirvijoki, Decker, Embréus and Fülöp, PRL **114**, 115002 (2015)]

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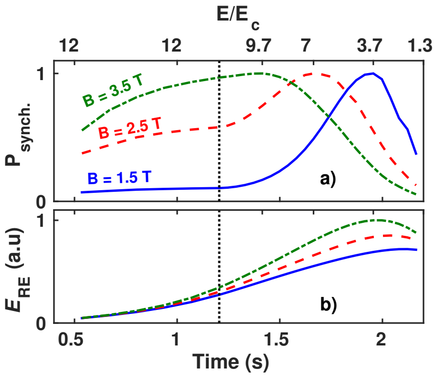
Runaway growth-to-decay transition

Synchrotron emission agrees with experiments!

- Emitted synchrotron power sensitive to particle energies and pitches
- Observed reduction is **not RE decay** but redistribution of REs in momentum space
- Runaways are still gaining energy when the emission declines



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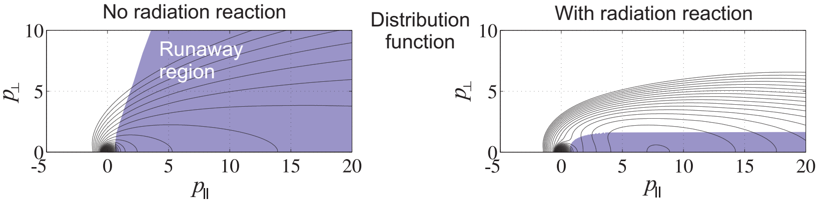
Qualitative effects of synchrotron radiation reaction

- Radiation reaction increases with perpendicular momentum.
- Runaway region shrinks to a region of small perpendicular momenta.
- Pitch-angle scattering of electrons out of the runaway region leads to an exponential decay of the electron distribution in the far-tail.
- Return fluxes of electrons into the runaway region due to collisional friction and radiation reaction can overcome the outflow due to pitch-angle scattering.



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Threshold for bump appearance

- Synchrotron radiation

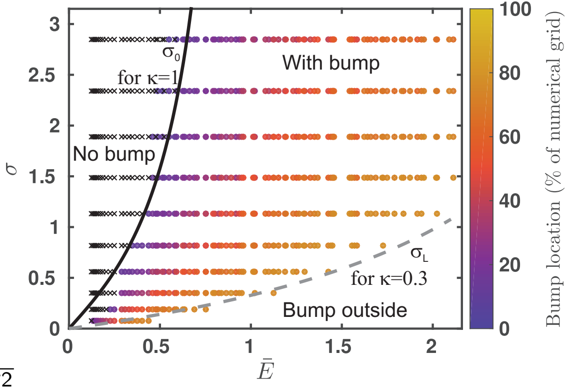
$$\sigma = \frac{\tau_c}{\tau_r} = \frac{2}{3\ln\Lambda}\frac{\Omega_e^2}{\omega_{pe}^2}$$

- Electric field

$$\bar{E} = \frac{E/E_c - 1}{2(1+Z_{\text{eff}})}$$

- Threshold for bump formation $\sigma > \sigma_0$

$$\sigma_0 = \frac{3\kappa/\bar{E} + \sqrt{8+\kappa^2/\bar{E}^2}}{2(\kappa^2/\bar{E}^2 - 1)}$$



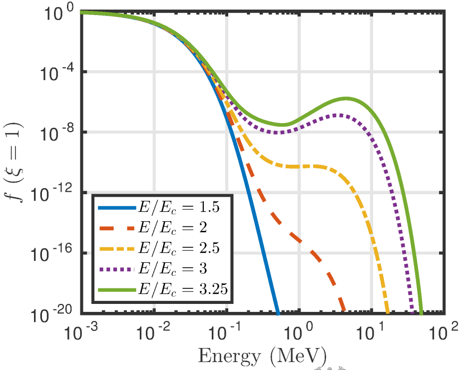
[Hirvijoki, Pusztai et al, to appear in JPP 2015]

Bump location in RE tail

Location of the bump

$$p_{\parallel b,\min} \simeq \frac{(E/E_c - 1)}{(1+Z_{\text{eff}})} \frac{(1+\sigma)}{\sigma}$$

- Bump location ∝ typical runaway energy.
- Bump location increases with the electric field, decreases with effective charge and decreases with synchrotron strength.



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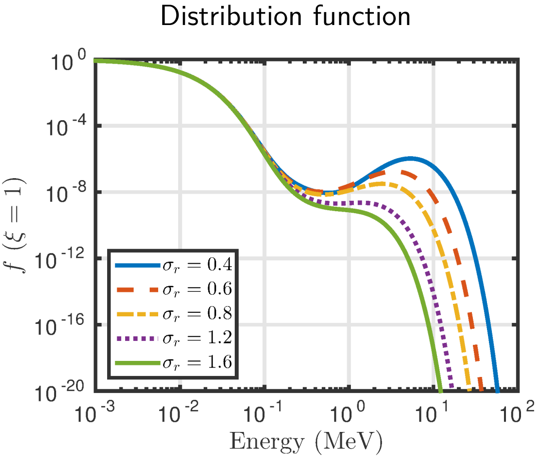
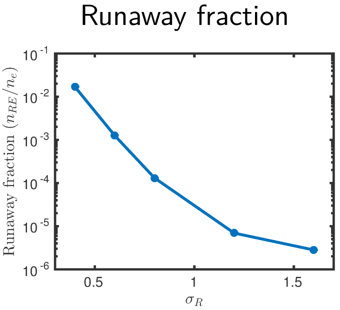
Synchrotron strength





Parameters:

$E/E_c = 3,\ Z_{\mathrm{eff}} = 1,\ T = 5$ keV

$$p_{\| b,\min} \simeq \frac{(E/E_c - 1)}{(1 + Z_{\mathrm{eff}})} \frac{(1+\sigma)}{\sigma}$$

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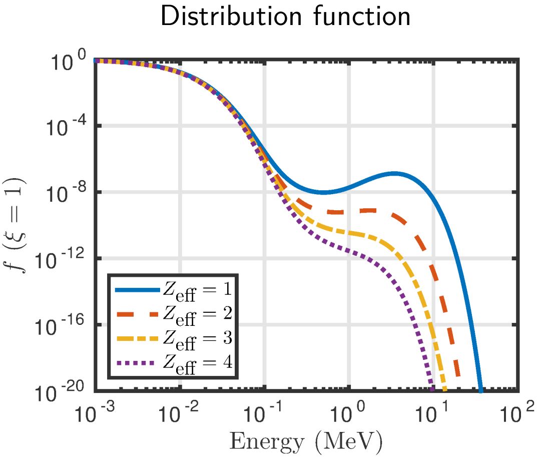
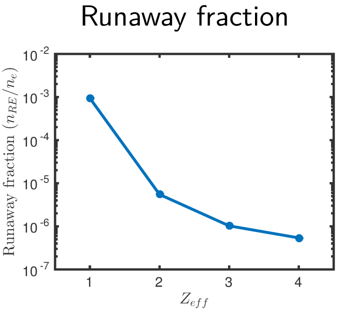
Effective charge



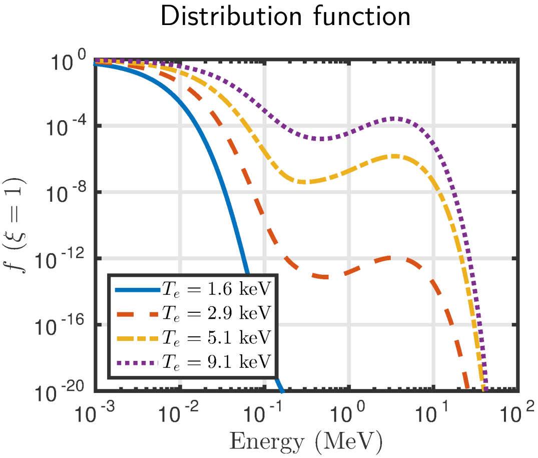


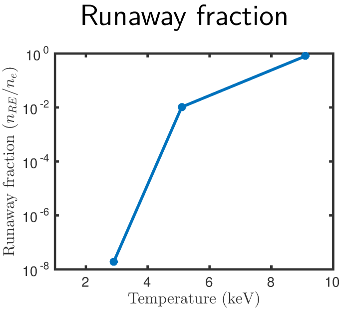
Parameters:

$\sigma = 0.6,\ E/E_c = 3,\ T = 5$ keV

$$p_{\|b,\text{min}} \simeq \frac{(E/E_c - 1)}{(1 + Z_{\text{eff}})} \frac{(1+\sigma)}{\sigma}$$

Electron temperature





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Outline

Fast-electron Bremsstrahlung radiation reaction

- Runaways experience inelastic collisions with both ions and thermal electrons
- Bremsstrahlung is emitted – radiation reaction effectively an isotropic slowing-down force
- Accounted for by a model operator,
  $C_{\mathrm{B}}^{(m)} = -\frac{\partial}{\partial \mathbf{p}} \cdot \left(\mathbf{F}_{\mathrm{B}}(\mathbf{p}) f_e(\mathbf{p}) \right)$,
  chosen to get correct energy moment:
  $F_{\mathrm{B}}(p) = -\sum_b n_b \int \mathrm{d}\sigma_{e\text{-}b}\, \hbar\omega$

How does Bremsstrahlung emission affect runaway dynamics?

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How does Bremsstrahlung emission affect runaway dynamics?

- Bremsstrahlung stopping power $< eE_c$ for energies below 100–200 MeV (for typical parameters)
- **Bremsstrahlung usually negligible** as often $E \gg E_c$ in disruptions

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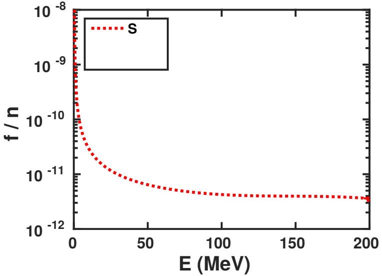
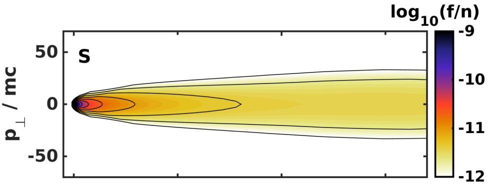
Effects of Bremsstrahlung radiation reaction





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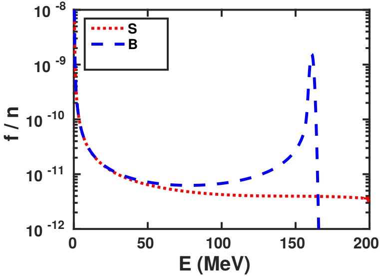

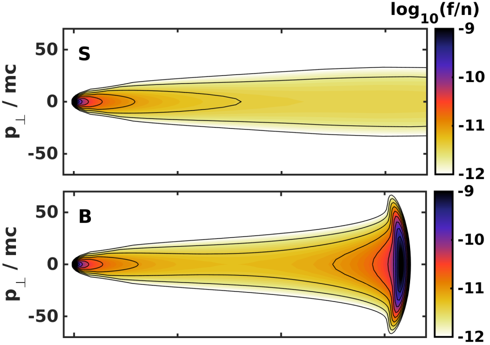

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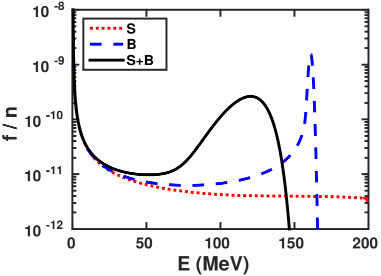

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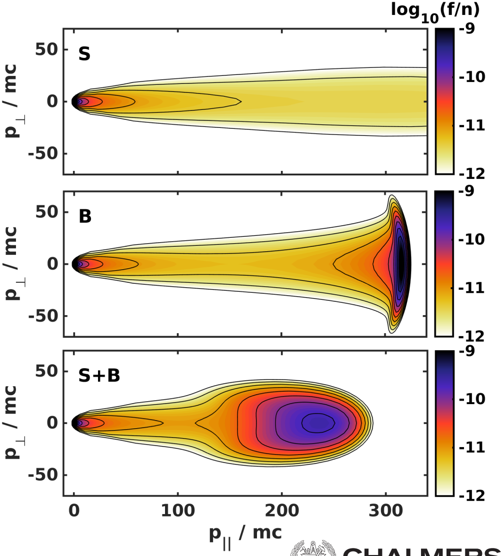

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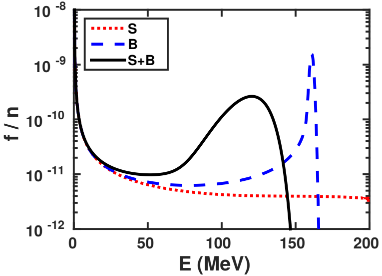

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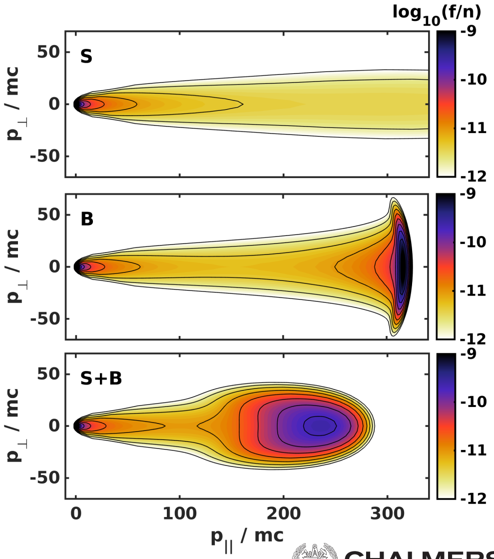
Bremsstrahlung increases pitch-angle scattering – can significantly affect the distribution function!



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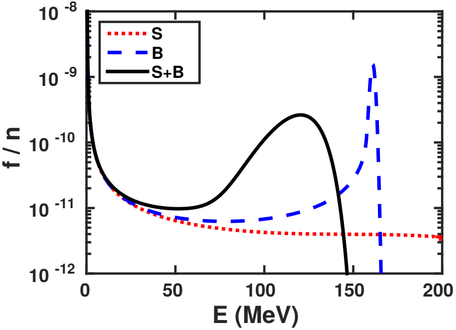

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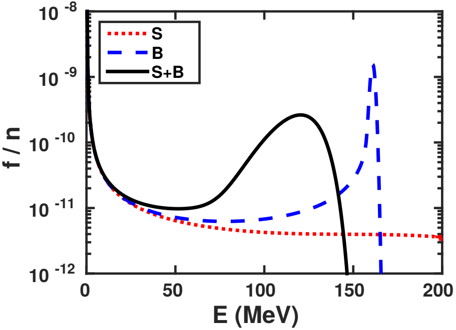

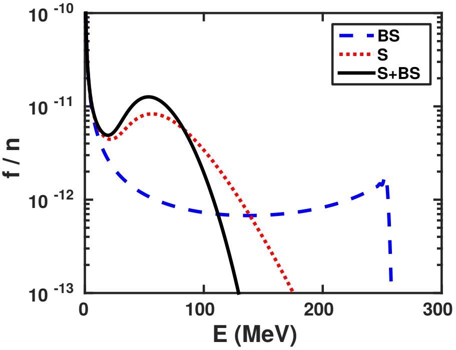

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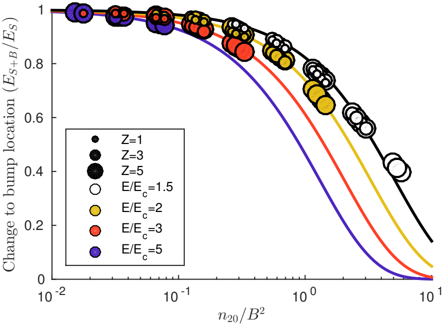
Bump location and energy spread



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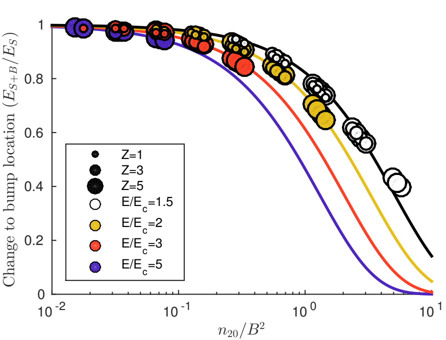

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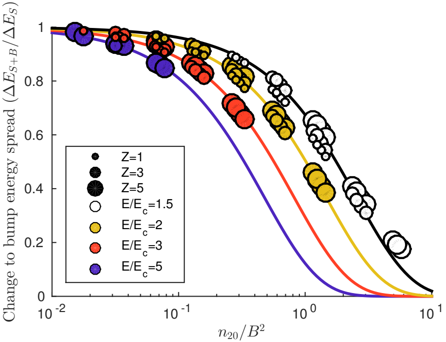
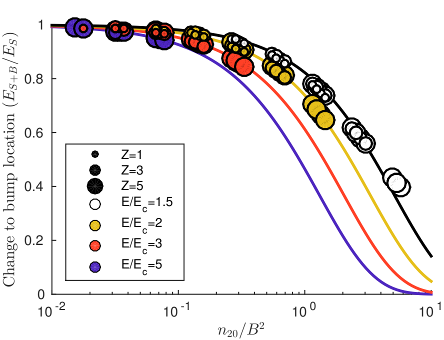
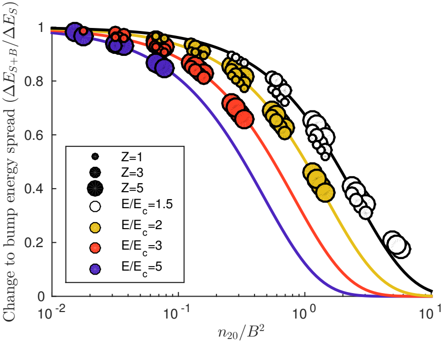


Bump energy spread

Bump location and energy spread



Bump location



Bump energy spread

Conclusion: Bremsstrahlung radiation moves the bump towards lower energies and less energy spread. The effect increases with n/B^2 and is not sensitive to the effective charge.

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Outline

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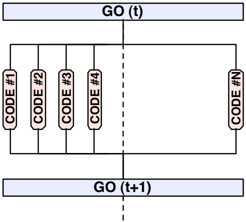
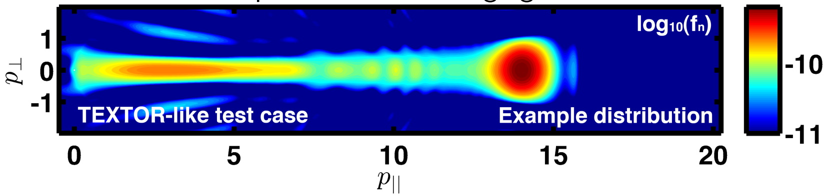
Towards self-consistency: GO+CODE

- GO evolves the global plasma parameters. CODE is evaluated in every radial grid point and GO time step.
- Each CODE call has its own set of numerical parameters (grid resolutions, iterations etc are independent).



Bumps due to the changing E-field!



[Papp et al, EPS 2015]

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Outline

Dynamics of runaway ions – Motivation

Motivation: Are runaway ions responsible for observed low mode number TAEs?

[Fülöp & Newton, PP **21**, 080702 (2014)]

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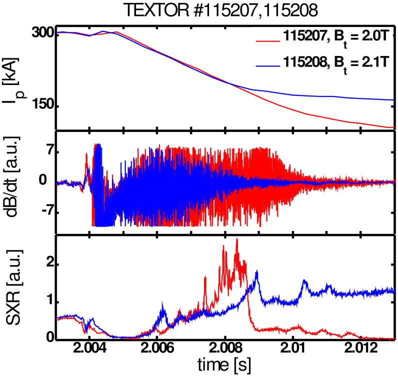
Dynamics of runaway ions – Motivation

Motivation: Are runaway ions responsible for observed low mode number TAEs?

[Fülöp & Newton, PP **21**, 080702 (2014)]

- 115207
  - $B = 2\ \mathrm{T}$
  - decrease in SXR signal
  - large magnetic fluctuations
  - no runaways
- 115208
  - $B = 2.1\ \mathrm{T}$
  - SXR signal increases
  - magnetic fluctuations disappear
  - runaways present

[Koslowski, EFDA project meeting 2012]



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Dynamics of runaway ions – CODION

- Largely analogous to electron runaway
- Use CODION to study ion distribution – adaptation of CODE
- Significant improvement over analytical models!

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Dynamics of runaway ions – CODION

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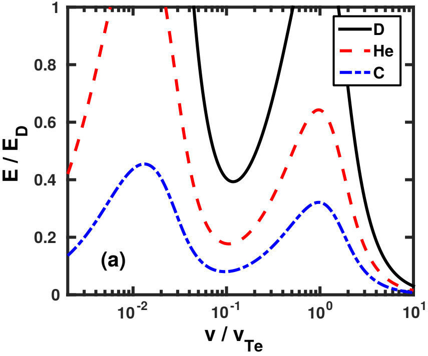

- Key difference to electron runaway: multiple peaks in friction force
- Direction of acceleration depends on Z/Z_{eff}

Parameters:

$n_C/n_D = 0.4\%$, $n_{He}/n_D = 5\%$, $Z_{\mathrm{eff}} = 1.2$

[Embréus, Newton, Stahl, Hirvijoki and Fülöp, Phys. Plasmas **22**, 052122 (2015)]

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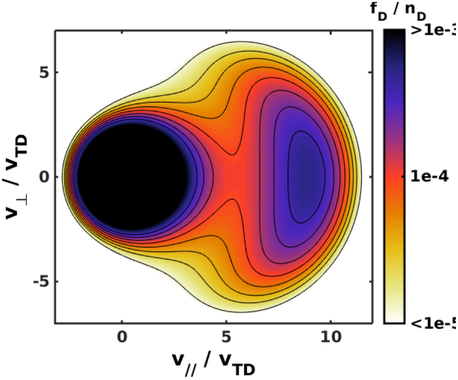
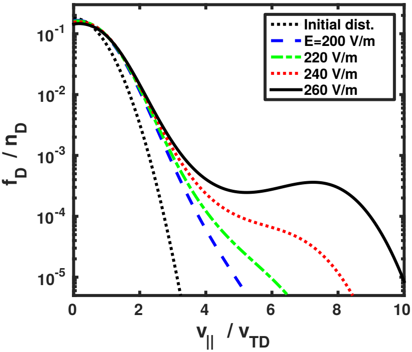
Dynamics of runaway ions – Results





- Typical runaway ion distribution exhibiting a large high-energy bump.
- D distribution after 2 ms of acceleration in disruption

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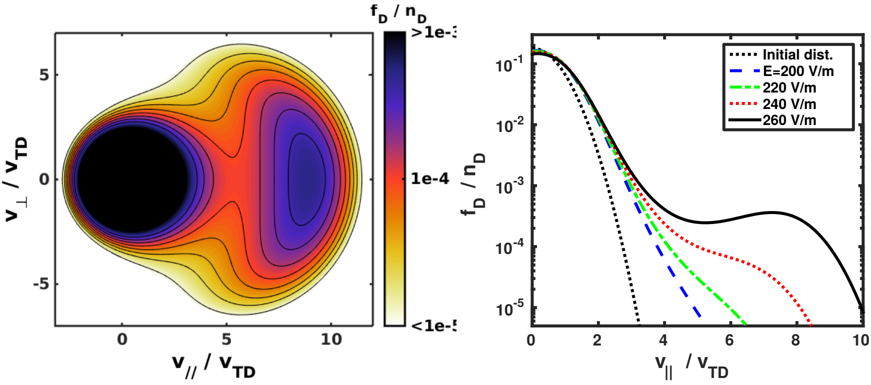
Dynamics of runaway ions – Results



- Typical runaway ion distribution exhibiting a large high-energy bump.
- D distribution after 2 ms of acceleration in disruption
- Here, $v_A/3 \sim$ 30-50 v_{TD}

Runaway ion energy too low to drive Alfvénic instabilities!

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Outline

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Conclusions

Elevated critical electric field can largely be explained by

- Temperature dependence and synchrotron radiation damping of RE growth rate
- Redistribution of electrons in momentum space (for E/E_c drop)

Synchrotron bump formation in the runaway tail

- Threshold condition and location for the bump

Bremsstrahlung moves the bump towards lower energies and less energy spread

- Effect increases with n/B^2 and is not sensitive to the effective charge.

Runaway ion dynamics

- Successfully treated numerically (CODION on github)

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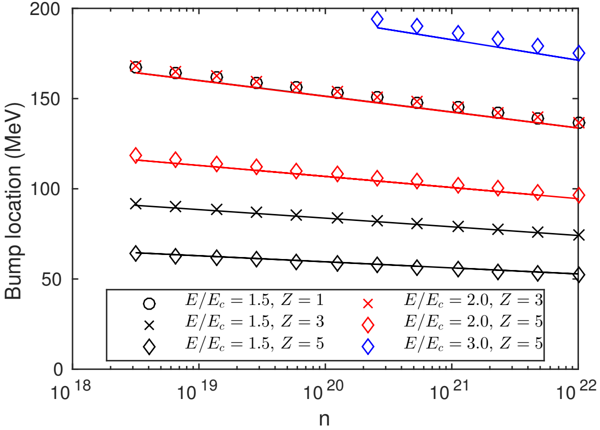
Recent papers

- CODE: [Landreman, Stahl and Fülöp, CPC **185**, 847 (2014)]
- Critical field:
  [Stahl, Hirvijoki, Decker, Embréus and Fülöp, PRL **114**, 115002 (2015)]
- Runaway ions: [Fülöp & Newton, PP **21**, 080702 (2014)],
  [Embréus, Newton, Stahl, Hirvijoki and Fülöp, PP **22**, 052122 (2015)]
- Synchrotron:
  [Hirvijoki, Pusztai, Decker, Embréus, Stahl and Fülöp, JPP (2015)],
  [Decker, Hirvijoki, Embréus, Peysson, Stahl, Pusztai and Fülöp, submitted to PPCF, arxiv.org/abs/1503.03881]
- EXEL-wave: [Pokol, Kómár, Budai, Stahl and Fülöp, PP **21**, 102503 (2014)]
- RMP: [Papp, Drevlak, Pokol and Fülöp, to appear in JPP (2015)]

Spare slides

Location of the bump induced by Bremsstrahlung radiation reaction



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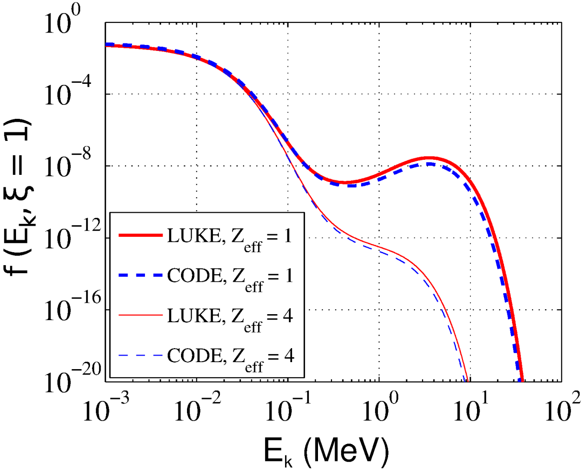
LUKE/CODE comparison



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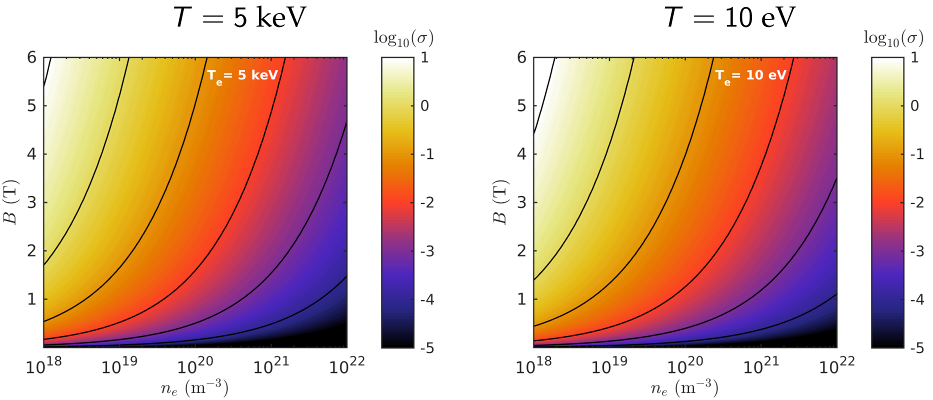
Synchrotron parameter σ



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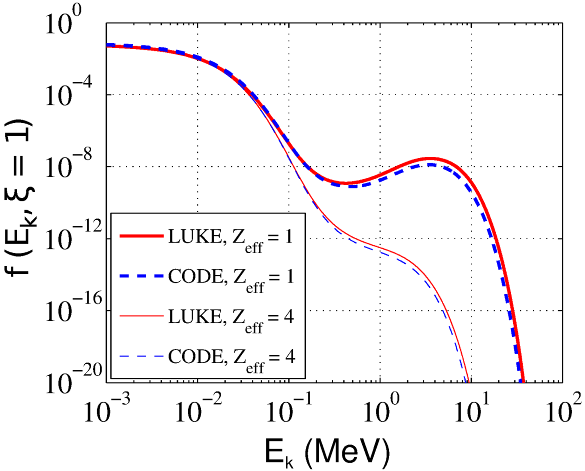

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Temperature evolution

- Energy balance equations for all species

$$\frac{3}{2}\frac{\partial(n_\mathrm{e}T_\mathrm{e})}{\partial t} = \frac{3n_\mathrm{e}}{2r}\frac{\partial}{\partial r}\left(\chi r\frac{\partial T_\mathrm{e}}{\partial r}\right) + P_\mathrm{OH} - P_\mathrm{rad} - P_\mathrm{ion} + P_\mathrm{c}^\mathrm{eD} + P_\mathrm{c}^\mathrm{eZ},$$

$$\frac{3}{2}\frac{\partial(n_\mathrm{D}T_\mathrm{D})}{\partial t} = \frac{3n_\mathrm{D}}{2r}\frac{\partial}{\partial r}\left(\chi r\frac{\partial T_\mathrm{D}}{\partial r}\right) + P_\mathrm{c}^\mathrm{De} + P_\mathrm{c}^\mathrm{DZ},$$

$$\frac{3}{2}\frac{\partial(n_\mathrm{Z}T_\mathrm{Z})}{\partial t} = \frac{3n_\mathrm{Z}}{2r}\frac{\partial}{\partial r}\left(\chi r\frac{\partial T_\mathrm{Z}}{\partial r}\right) + P_\mathrm{c}^\mathrm{Ze} + P_\mathrm{c}^\mathrm{ZD}.$$

- Energy exchange in collisions: $P_\mathrm{c}^{kl} = \frac{3}{2}\frac{n_k}{\tau_{kl}}(T_l - T_k)$
- Radiation: $P_\mathrm{rad} = P_\mathrm{Br} + \sum_i P_{\mathrm{line},i}$, and $P_{\mathrm{line},i} = n_i n_\mathrm{e} L_i(n_\mathrm{e}, T_\mathrm{e})$.
- Impact ionization and radiative recombination determine n_i:

$$\frac{dn_i}{dt} = n_\mathrm{e}(I_{i-1}n_{i-1} - (I_i + R_i)n_i + R_{i+1}n_{i+1})$$

- Requires externally provided neutral impurity profile.

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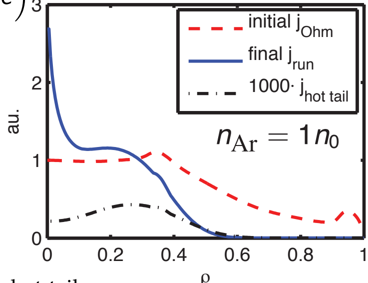
Induction equation

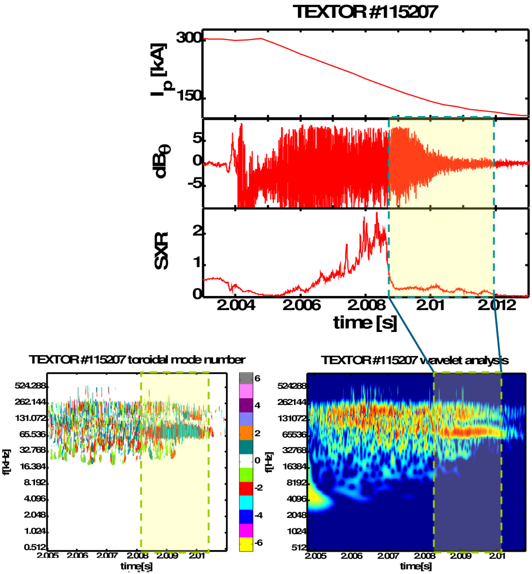
- Electric field is induced to keep current constant

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E}{\partial r}\right) = \mu_0 \frac{\partial}{\partial t}\left(\sigma_{\parallel} E + n_{\mathrm{r}} ec\right)$$

- Instead of modelling the velocity space dynamics for the electrons that are already inside the runaway region, we only consider their total density.



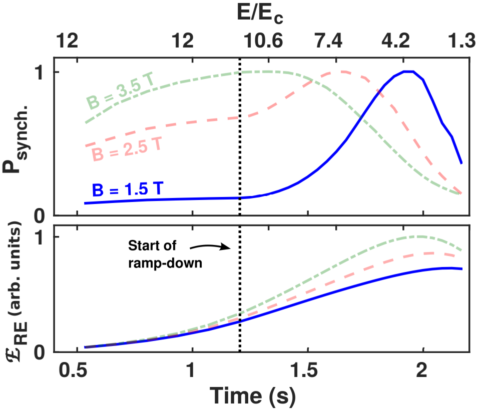
$$\frac{\partial n_{\mathrm{r}}}{\partial t} = \left(\frac{\partial n_{\mathrm{r}}}{\partial t}\right)^{\mathrm{Dreicer}} + \left(\frac{\partial n_{\mathrm{r}}}{\partial t}\right)^{\mathrm{hot\text{-}tail}} + \left(\frac{\partial n_{\mathrm{r}}}{\partial t}\right)^{\gamma} + $$
$$+ \left(\frac{\partial n_{\mathrm{r}}}{\partial t}\right)^{\mathrm{avalanche}} + \frac{1}{r}\frac{\partial}{\partial r} r\, D_{\mathrm{RR}} \frac{\partial n_{\mathrm{r}}}{\partial r}.$$



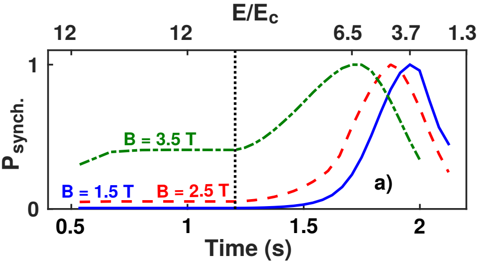
[Koslowski, EFDA project meeting 2012]

Runaway growth-to-decay transition

Rosenbluth-Putvinski



No avalanche



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Synchrotron radiation reaction

- Radiation reaction force leads to a flow towards lower particle momenta and smaller pitch-angles
- Reduces runaway rate
- Can lead to bump formation in RE tail

Without radiation reaction **With radiation reaction**